TESTING THE FOUNDATIONS OF RELATIVITY USING CRYOGENIC OPTICAL RESONATORS

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We describe three different tests of Relativity, a test of the isotropy of the speed of light, a test of the independence of the speed of light on the velocity of the laboratory, and a test of the gravitational clock shift for an electronic clock. The three tests, using cryogenic optical resonators (COREs) have yielded improvements of the best previous tests by up to a factor of three. The potential for future improvements is discussed.

1. Introduction

One of the outstanding challenges of physics is a unified theory of all fundamental forces. So far, no simple way of unifying the theory of gravity and quantum mechanics has been found. Indeed, approaches to this problem, e.g. quantum gravity theories, are found to lead to violations of the basic principle on which General Relativity rests, namely the Einstein Equivalence Principle (EEP). [1] This situation motivates significantly improved experimental tests of various aspects of the EEP, performed both on Earth and in space.

The EEP consists of three principles,[2] the weak equivalence principle, the principle of local position invariance (LPI) and the principle of local Lorentz invariance (LLI), i.e. the validity of Special Relativity for local experiments. In this report, we shall discuss a test of one aspect of LPI, namely the universality of the gravitational clock shift, and two aspects of Special Relativity (SR), commonly summarized as the constancy of the speed of light. A part of the results has been presented in detail elsewhere.[3]

1.1 Tests of Special Relativity

Two of the three principles of SR may be tested as follows. Consider a hypothetical preferred frame, in which the speed of light is isotropic and has the value c_0, and which is usually identified with a frame of isotropic microwave background radiation. If SR holds, in any inertial frame moving at

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an arbitrary velocity $\mathbf{v}$ relative to the preferred frame, experiments will find

(i) isotropy: a speed of light that is independent of the angle θ between the propagation direction and $\mathbf{v}$ ("Michelson-Morley experiment" [4]),

(ii) velocity independence: this speed of light has a value independent of the actual value of the velocity v, and equals c_0 ("Kennedy-Thorndike experiment" [5])

The third principle, time dilation, requires experiments linking two inertial frames; e.g. relativistic Doppler shift measurements.[6]

The tests can be described within the framework of the Mansouri-Sexl (MS) test theory.[7] Concerning (i) and (ii), deviations from SR are parametrized as

$$\frac{c(v,\theta)}{c_0} = 1 + A\frac{v^2}{c_0^2} + B\frac{v^2}{c_0^2}\sin^2\theta \ . \tag{1}$$

The parameters A and B are related to the MS-transformation parameters by $A = \alpha - \beta + 1$, and $B = \beta - \delta - 1/2$. In nonrelativistic physics, they would be on the order of unity, in SR they vanish. The velocity $\mathbf{v}$ is taken as the velocity with respect to the microwave background. The best tests so far, performed by Brillet and Hall [4] and Hils and Hall [8] have yielded $|B| < 5\cdot 10^{-9}$ and $|A| < 7 \cdot 10^{-5}$, respectively. The limit for A is by far weaker than those found in the isotropy and time dilation tests. Thus, the overall accuracy of SR is currently limited by the velocity invariance test.

An apparatus suitable for tests of isotropy and velocity independence is an electromagnetic resonator and a frequency measurement device. In the case of the velocity independence test, this latter device must not be based on a length standard. The resonator defines a propagation direction (and thus θ) for electromagnetic modes. The boundary conditions require that the resonance frequenices are proportional to the speed of light for the particular propagation direction, $\nu \sim c(v,\theta)$. The tests consist in measuring the frequency as a function of orientation θ and laboratory velocity v, analyzing the frequency according to Eq.(1), and setting upper limits for $|A|$ and $|B|$. A change in orientation can be brought about either by actively rotating the set-up (as in the experiment of Brillet and Hall), or by making use of the daily rotation of the Earth. To obtain a change in velocity v, one must rely (in the case of laboratory experiments) on the Earth's motion. As shown in Fig. 1, the velocity v is modulated on a daily and yearly scale, with amplitudes differing by a factor of about 100.

1.2 The gravitational clock frequency shift

The frequencies of a clock placed at two points x_0 and x_1 are related by

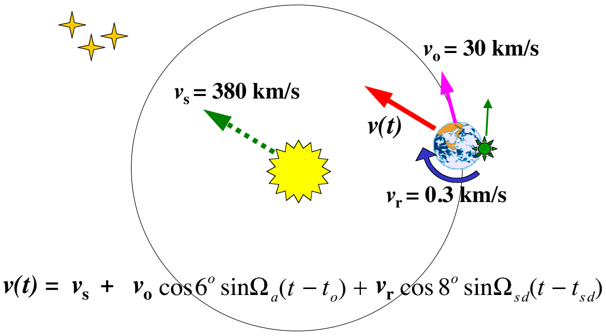


Figure 1. Schematic of the motion of the laboratory with respect to the microwave background with contributing velocities (at the latitude of Konstanz). $2\pi/\Omega_{sd} = 1$ sidereal day and $2\pi/\Omega_a = 1$ year.

$$\nu(x_1) = \nu(x_0)\left(1 + \alpha_{clock}\frac{U(x_1) - U(x_0)}{c^2}\right) ,$$

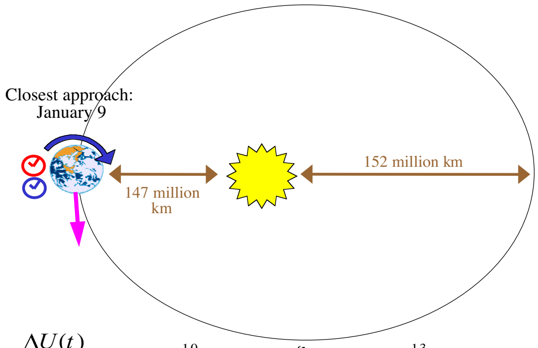
where $U(x)$ is the Newtonian gravitational potential at location x, and α_{clock} is a dimensionless parameter that describes the coupling of the energy levels of the clock to gravity. If LPI holds, α must be equal for all types of clock (provided they are based on non-gravitational effects); General Relativity further states that $\alpha_{clock} = 1$. In alternative theories, α may depend on the type of clock.

Various experiments have been performed to test the values of α_{clock} for different types of clocks.[2] To our knowledge, however, no accurate test of the clock shift of an clock based on an electronic transition ("electronic clock") has been performed so far.

Some experiments compared clocks placed at different locations that communicated the frequency information.[9,10] Alternatively, in null experiments clocks of different type are moved together in a gravitational potential while their frequencies are compared over time.[11,12] Here, the shifts $\Delta\nu = \nu(x_1) - \nu(x_0)$ in their frequencies when moved together from location x_0 to x_1 are related by

$$\frac{\Delta\nu_1}{\nu_1(x_0)} - \frac{\Delta\nu_2}{\nu_2(x_0)} = (\alpha_1 - \alpha_2)\frac{U(x_1) - U(x_0)}{c^2} .$$

If the two clock frequencies are close, $\nu_1(x_0) \approx \nu_2(x_0) = \nu$, then the left hand side is approximately equal to the relative beat frequency variation $\Delta\nu_b/\nu$, where $\nu_b(x) = \nu_1(x) - \nu_2(x)$. For a null experiment performed on Earth, one again makes use of the motion of the Earth, see Fig.2, so that $x_1 = x_1(t)$. Earth's rotation and its elliptical orbit give rise to a solar gravitational potential value on the Earth that has a daily and yearly modulation, differing by a factor $\simeq 1000$ in amplitude.



$$\frac{\Delta U(t)}{c^2} = 3.3\cdot 10^{-10}\sin(\Omega_a t-\xi)+3.2\cdot 10^{-13}\sin(\Omega_{sol} t-\theta)$$

Figure 2. The Earth's motion leads to a time-dependence of the sun's gravitational potential U on the Earth's surface with components of period $2\pi/\Omega_a$ and $2\pi/\Omega_{sol}$ =1 solar day.

2.1 Test of velocity invariance and gravitational shift for an electronic clock

In an experiment performed in 1998,[13] the frequencies of a CORE-stabilized laser and an iodine (I_2)-stabilized laser where compared during a period spanning 190 days. The beat frequency data ν_b was analyzed for pure velocity invariance violation and pure LPI violation. The fits and errors are shown in Figs. 3 and 4. The results can be summarized as $A = (1.9\pm2.1)\cdot10^{-5}$, and $|\alpha_{elec} - \alpha_{res}| = (2.2 \pm 1.6)\%$, respectively. The uncertainty in the value of A is three times lower than the previous best experiment,[8] and improves the overall accuracy of the experimental verification of SR by the same factor. With the known limits for resonators,[11] we can deduce a limit for a LPI

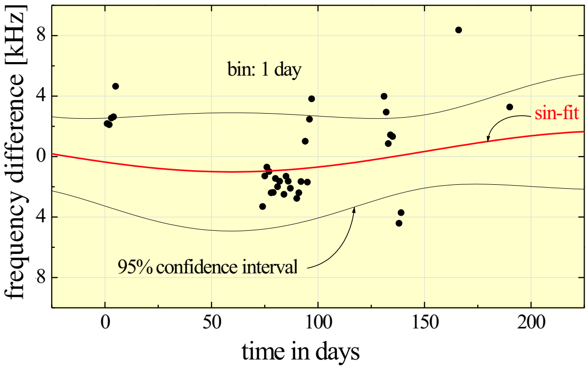


Figure 3. Fit of a velocity violation signal to the data of a iodine - CORE comparison.

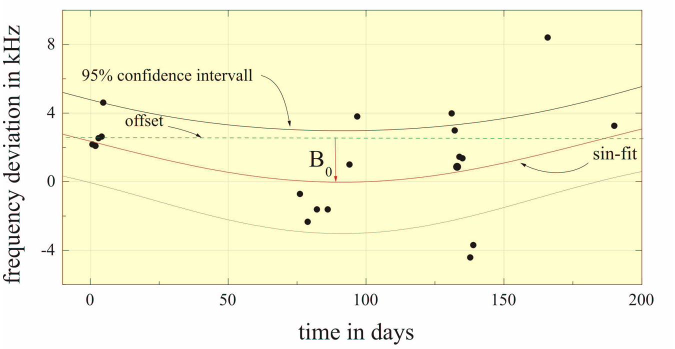


Figure 4. Fit of an LPI violation signal to the data of a iodine - CORE comparison.

violation of electronic clocks of approximately $|\alpha_{elec} - 1| < 4\,\%$.

2.2 Isotropy test

The set-up of an isotropy test currently in progress is shown in Fig.5. The use of two orthogonal cavities has the advantage of doubling the hypothetical signal amplitude. The signal period is 12 h. Denoting the frequencies of the two cavity-locked lasers by ν_1 and ν_2 and their beat frequency change by $\Delta\nu(\theta) = \Delta\nu_1(\theta) - \Delta\nu_2(\theta)$, the isotropy violation signal is

$$\frac{\Delta\nu(\theta)}{\nu} \approx \frac{\Delta\nu_1}{\nu_1} - \frac{\Delta\nu_2}{\nu_2} = \frac{c(v,\theta)}{c_0} - \frac{c(v,\theta+\pi/2)}{c_0} = -B\frac{v^2}{c_0^2}\cos 2\theta \ . \tag{2}$$

The laser beams are coupled to the cavities through optical windows, without any beam direction stabilization. Reflection Pound-Drever-Hall locks are used, with $3\,f_m$ detection. The powers transmitted through the cavities are below 100 nW. The power through one of the cavities is stabilized by

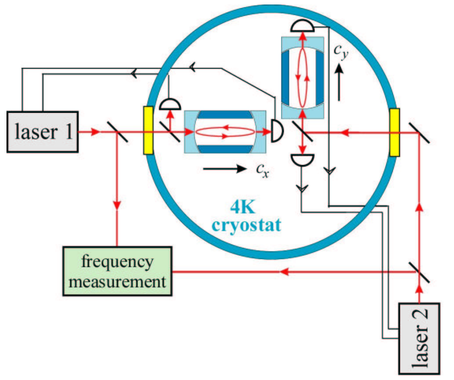


Figure 5. Set-up for the isotropy test. 1064 nm Nd:YAG lasers are used.

feeding back to the pump laser diode. The free-space coupling of the two beams implies that a relative shift of the beams and the cavities will lead to shifts in the locked frequency, due to spurious etalons. Such shifts occur due to evaporation of the cryogenic liquids, especially during liquid He refills. The refills also seem to change the mechanical stresses on the resonators, leading to frequency shifts.

Beat data was collected during nights and week-ends over a period spanning 6 weeks, so individual data blocks are typically half a day to a few days long. For each data block, we remove the linear drift and fit to Eq. (2). An example is shown in Fig. 6. Analyzing 25 records, we find a beat modulation amplitude of $(-0.39 \pm 0.58)\,\mathrm{Hz}$, which implies $B = (0.87 \pm 1.3) \cdot 10^{-9}$. This limit has an uncertainty about two times smaller than the previous best test.[4]

3. Summary and future prospects

Our experiments provide new limits on violations of the Einstein equivalence principle. A comparison of a CORE with a iodine frequency reference, yielded a test of the independence of the speed of light on the laboratory velocity with a three-fold improvement in accuracy. This result also improves the overall accuracy of the verification of Special Relativity by a factor of 3. Alternatively, we may interpret the experiment as a null gravitational redshift experiment which tested the principle of local position invariance, as applied to length-based and electronic transition-based clocks, at the 4 % level. Both

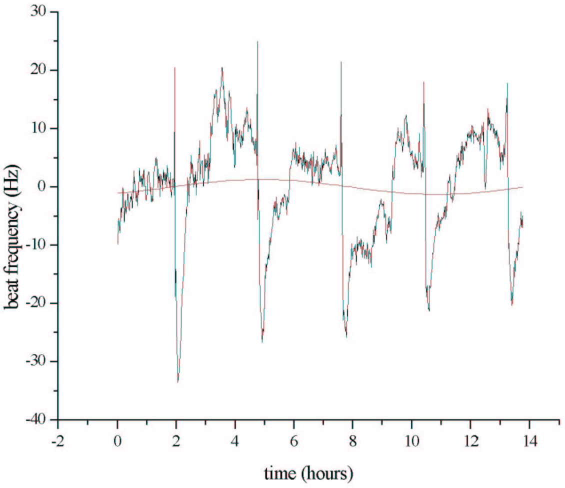


Figure 6. Beat frequency between two orthogonal COREs: typical data block after subtraction of linear drift and fit. The periodic spikes are due to the liquid nitrogen refills.

tests made use of the large modulation amplitude of Earth's orbital motion around the Sun. A two-cavity Michelson-Morley test is currently underway and has yielded, as a preliminary result, a twofold improvement in accuracy.

Our tests were limited by effects that can be eliminated with improved set-ups. Concerning the feasible performance of the COREs, we show in Fig. 7 the results of a beat between a fiber-coupled CORE and a previous version of a free-spaced coupled CORE, each housed in a separate cryostat.[14] The beat does not show any spikes due to refills. The beat change after an accidental bump to one of the cryostats puts in evidence the mechanical sensitivity of the apparatus. The drift before and after the "bumping" is at the 0.1 Hz/s level. Based on the instability of $2 \cdot 10^{-15}$, i.e. 0.6 Hz for averaging times on the order of 15 min to an hour, a two-cavity isotropy experiment with a rotation period of 15 min should allow a forty-fold improvement of the present accuracy.

In the near future, the instability of the electronic lock will not be a limiting factor. We have studied the stability of the locks by locking two lasers to adjacent longitudinal modes of a single CORE. Free-space coupling and copropagating waves were used, and a combination of 3 f_m-reflection lock and a f_m-transmission lock. The beat had a linewidth below 1 Hz, remained within 5 Hz for 12 h, and refill peaks were almost absent, thanks to the transmission lock. As Fig. 7 shows, the Allan standard deviation of the beat (not normalized to a single laser) falls to $4 \cdot 10^{-16}$ at 5 min, and is therefore sufficiently below the level achieved at present in a two-cavity comparison.

A new generation of velocity invariance and gravitational clock shift uni-

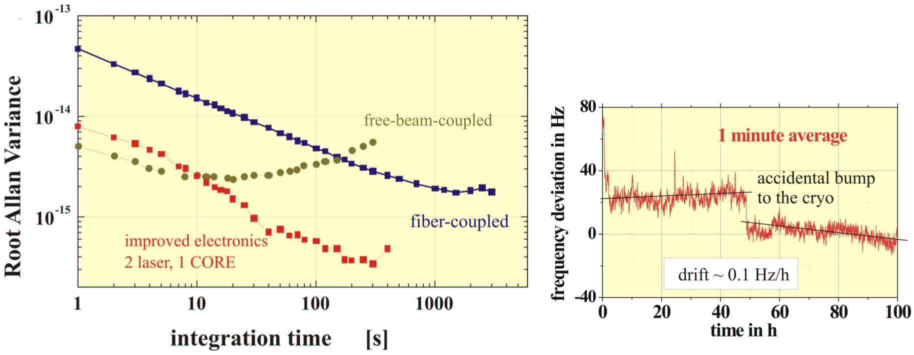


Figure 7. Left: Top line: frequency instability of CORE, deduced from the comparison between a fiber-coupled and a free-space coupled system, shown on the right. The line marked "free-beam coupled" is for a different set-up using free-space coupled systems only, and in which the incoupling was higher and the locks were optimized for good short-term stability. Line marked "2 Laser 1 CORE" shows the lock instability, deduced from the beat between two free-space coupled lasers locked to a single resonator.

versality tests could be performed using femtosecond comb techniques.[15] An accurate comparison of an optical cavity frequency with a microwave standard is possible with reasonable effort. Microwave oscillators with instability at the level of that of our best CORE results and on the relevant time scale of one day do exist, e.g. microwave ion traps [16] or hydrogen masers stabilized to the international time scale via GPS. Assuming a combined CORE plus microwave clock instability of $5 \cdot 10^{-15}$ at one day, a three-month comparison could improve the velocity invariance test accuracy by a factor of 100.

Further improvements of the LPI test for *electronic* clocks presented here are likely to come from the frequency measurements of various optical frequency standards (Calcium atoms, Mercury ion) underway in national laboratories [17]. However, the data obtained in the velocity invariance test could be used to impose new limits on the nonuniversality of gravitational clock shift for *resonators*, which so far stands at $1.7 \cdot 10^{-2}$ for microwave resonators.[11]

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