Highly sensitive silicon crystal torque sensor operating at the thermal noise limit

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We describe a sensitive torque detector, based on a silicon single-crystal double-paddle oscillator (DPO). The high Q-factor ($\sim 10^5$ at room temperature and in vacuum) makes DPOs well suited for the detection of weak forces. The limiting sensitivity of a sensor is given by Brownian (thermal) noise if all external disturbances are eliminated. In this case, the minimum detectable force can be decreased by measuring over a time significantly longer than the oscillator's relaxation time. We demonstrate operation in this regime, with integration times of up to 14 h. A resulting torque sensitivity of 2×10^{-18} N m is reached. Tests are performed to show that the sensor is only affected by thermal noise. The present sensor is well suited for measurements of extremely weak forces, e.g., of gravitational attraction between laboratory masses. © *2007 American Institute of Physics.*
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I. INTRODUCTION

One of the first measurements of the Brownian motion of a mechanical oscillator goes back to Gerlach in 1927, who studied the torsional Brownian noise of a small mirror attached to a very fine wire.[1] A theoretical analysis of this phenomenon was provided by Uhlenbeck and Goudsmith in 1929.[2] A deeper understanding of the origin of Brownian noise was developed by Nyquist, who pointed out the existence of a connection between stochastic motion and the internal mechanical loss of the material.[3] A further generalization of this concept was given by Callen and Welton, through the fluctuation-dissipation theorem.[4]

In the last decade the relevance of mechanical oscillators for precision measurements of weak forces has steadily grown.[5] The detection of the Casimir force,[6] 3D microscopy with subnanometer resolution,[7] and attogram mass detection[8] are some of the most recent examples of the results achieved by the use of mechanical oscillators in high-precision experiments. Particularly challenging applications of ultrasensitive force sensors are tests of Newton's law at small distance[9–14] and optical measurements of small displacements.[15,16] In many cases, Brownian noise of the detector represents the desirable ultimate limit to their sensitivity. A review of the measurement and data analysis strategies, developed to improve the sensitivity of these detectors, can be found in the work by Gillies and Ritter.[17]

In the present work we report on a torque sensor based on a single-crystal silicon oscillator, the double-paddle oscillator (DPO). This sensor type, originally developed to measure internal friction of thin films,[18,19] is very well suited to the measurement of weak forces because of its high Q-factor. Under vacuum operation and at room temperature, it is possible to detect and characterize its Brownian motion, which is due to the scattering of phonons.

The article is structured as follows. We briefly review the theory of Brownian motion for a torsional oscillator, relevant for comparing the measured force sensitivity with the thermal noise limit. Based on this theory we discuss the measurement procedure for detection of a weak time-harmonic force in the presence of stochastic noise. Then, we review the main properties of the microfabricated DPOs and describe the experimental apparatus. The experimental results are given in the last section, which also describes tests that demonstrate that the measured noise is consistent with Brownian noise.

II. THEORY OF BROWNIAN NOISE

A simple model can describe the angular fluctuations of a torsional oscillator due to Brownian noise and is suitable for characterizing the sensitivity of a variety of precision experiments, e.g., weak force sensors and gravitational wave detection.[20]

The equation of motion of a harmonic torsional oscillator driven by Brownian noise is

$$I\ddot{\vartheta} + \beta\dot{\vartheta} + D\vartheta = M(t), \tag{1}$$

where I is the moment of inertia around the torsion axis, ϑ is the angular deflection of the oscillator, β is the damping coefficient, D is the spring constant, and M is a fluctuating torque. Equation (1) is a Langevin equation for a simple harmonic oscillator of frequency $\omega_R^2 = D/I$. We assume that $M(t)$ has the following properties:

(i) zero mean value;
(ii) its variance is a constant in time: $\overline{M^2(t)} = \text{const}$; and
(iii) its values at two different times are uncorrelated.

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The angular fluctuations of an oscillator excited by such a stochastic torque do not obey the statistics of pure random noise. This is a consequence of the correlations introduced by the oscillator. If the oscillator's deflection at time t_0 is $\vartheta(t_0)$, the probability distribution of its deflection at a later time t is given by[21]

$$P[\vartheta(t)|\vartheta(t_0)] = \frac{\vartheta(t)}{\overline{\vartheta^2}(1-e^{-2t/\tau})} I_0\left(\frac{\vartheta(t)\vartheta(t_0)e^{-t/\tau}}{\overline{\vartheta^2}(1-e^{-2t/\tau})}\right) \exp\left[-\frac{\vartheta^2(t)+\vartheta_0^2(t)e^{-t/\tau}}{2\overline{\vartheta^2}(1-e^{-2t/\tau})}\right], \quad (2)$$

where $\tau=2I/\beta$ is the mechanical relaxation time of the oscillator, $\overline{\vartheta^2}$ is the mean square deflection, and I_0 is the modified Bessel function.

If the oscillator is excited by an external harmonic torque at the oscillator's resonance frequency, $\Gamma=\Gamma_0 \sin(\omega_R t)$, the minimum detectable torque amplitude can be derived using Eq. (2) and is given by[21–23]

$$(\Gamma_0)_{\min} = \pi\sqrt{\frac{k_B T I}{\tau \Delta t}} = \pi\sqrt{\frac{k_B T I \omega_R}{2Q\Delta t}}, \quad (3)$$

if the measurement time Δt is smaller than the oscillator relaxation time τ. Here, the Q-factor has been introduced through $\tau=2Q/\omega_R$. The improvement of torque sensitivity with increasing measurement time can easily be pictured. During the measurement time the response of the oscillator to the external force increases steadily due to the phenomenon of resonance, whereas the response due to Brownian noise fluctuates. This is the regime in which the most sensitive torsion pendula operate,[23] whose relaxation times are of the order 10^6–10^9 s. A detailed analysis of the sensitivity of torsion pendula can be found in the book by Chen and Cook.[24]

Increasing the measurement time Δt beyond τ does not lead directly to an improved torque sensitivity unless an appropriate data analysis is performed.[5,24] Following Uhlenbeck and Goudsmit,[2] it is necessary to develop the measured angular displacement in a Fourier series, $\vartheta(t)=\Sigma_k \vartheta_k(t)$, where the index $k=0,\ldots,\infty$ denotes the frequency harmonics $\omega_k=2\pi k/\Delta t$. We first consider the oscillator motion in the absence of external torque. The term k' of the series at the oscillator's resonance frequency is the one of interest. The time average of $(\vartheta_{k'}(t))^2$ depends on the measurement time as

$$\overline{\vartheta_{k'}^2} = \frac{4k_B T Q}{I\omega_R^3 \Delta t}. \quad (4)$$

Both quadrature amplitude of $\vartheta_{k'}$ then also average to zero, as $(\Delta t)^{-1/2}$ for $\Delta t \gg 4Q/\omega_R$.

A similar result can be obtained by applying Nyquist's theorem.[20] The potential energy of the oscillator, in the absence of external excitation, can be calculated from Eq. (4) and, as shown in Ref. 2, is constant and independent of the observation time as expected from the equipartition theorem.

The signal-to-noise ratio of a measurement of a resonant torque of amplitude Γ_0 is defined as the ratio between the corresponding steady-state oscillator amplitude and the Brownian noise amplitude given by Eq. (4). Setting this ratio equal to unity yields the minimum detectable torque in the case $\Delta t \gg \tau$,

$$(\Gamma_0)_{\min} = \sqrt{\frac{4k_B T I \omega_R}{Q\Delta t}}. \quad (5)$$

Thus, the minimum detectable torque decreases with the square root of the measurement duration. The validity of this analysis is limited to the case of noise with white spectrum. An example of weak (gravitational) force detection using detection of the oscillator amplitude at the resonance frequency is given in Ref. 9.

It is interesting to consider the statistical properties of the oscillator's response. In the following analysis we assume that the deflection of the oscillator is measured by a lock-in technique where the local oscillator is tuned to the oscillator's resonance frequency. This yields the slowly varying amplitude $r(t)$ and phase $\psi(t)$ of the oscillator's deflection $\vartheta(t)=r(t)\cos(\omega_R t-\psi(t))$. The quadrature amplitudes $X(t)=r(t)\cos\psi(t)$ and $Y(t)=r(t)\sin\psi(t)$ can then be calculated. In steady state, the probability distribution function for these two quantities is given by $W(X,Y)=W(X)W(Y)$, where[25]

$$W(X) = \left(\frac{I\omega_R^2}{2\pi k_B T}\right)^{1/2} \exp\left(-\frac{I\omega_R^2}{2k_B T}X^2\right). \quad (6)$$

From Eq. (6) it follows that both quadratures have vanishing mean value, while their variance is equal to $k_B T/I\omega_R^2$, as expected from the equipartition theorem.

III. EXPERIMENTAL RESULTS

A. Description of apparatus

Single-crystal silicon has low internal friction and a large knowledge base exists concerning its fabrication into appropriate geometries. These two properties make it the favorite material for mechanical sensors in many research fields. Our oscillator was developed for an experiment to detect gravity at short (<1 mm) distances.[14] The design we used was developed by Kleiman and co-workers[18] and later improved by Pohl and co-workers,[19] who used it for characterizing the elastic properties of thin films. The oscillator is shown in Fig. 1. It was fabricated from a 300 μm thick, float-zone refined, double-side polished, ⟨100⟩-oriented, p-doped silicon wafer with a room-temperature-specific resistance larger than 10 kΩ cm. The fabrication procedure was developed in our group and is based on wet etching.[26] The sensor consists of two masses, head and wings, connected by a torsion rod, the neck. The wings are themselves connected to a base (foot) by a thinner rod, the leg. The vibrational modes of this structure have been fully characterized in the range between 0.1 and 10 kHz.[19] In the present work a torsional mode, denoted by AS2 in the literature, was used. In this mode the head oscillates, twisting the neck, while the wings' motion is out of the oscillator's plane around an axis orthogonal to the neck length. The resonance frequency ν_R of the AS2 mode of the oscillator was $\nu_R=(5921.303\pm0.003)$ Hz and its quality factor was Q

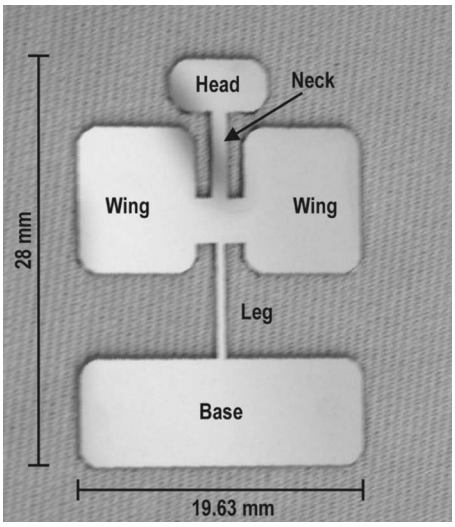

FIG. 1. The single-crystal silicon double-paddle oscillator used in this work. The thickness is approximately 300 μm.

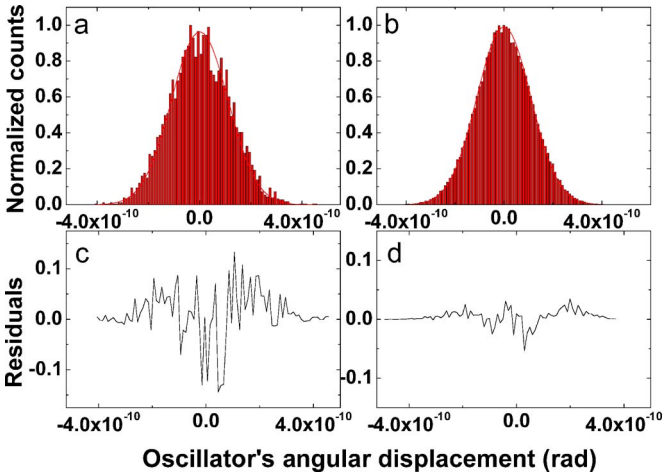

FIG. 2. Histograms of the oscillator's X angular displacement quadrature measured at resonance and in the absence of external excitation for two different measurement durations: (a) 2.5×10^3 s and (b) 1.7×10^5 s. The continuous curves represent Gaussian fits. (c) and (d) show the fit residuals.

$=(1.86\pm0.02)\times10^5$, which corresponds to a relaxation time $\tau=10$ s. The full width of the resonance curve at half-power was $\Delta\nu=\nu_R/Q=0.033$ Hz.

In our experiment the oscillator's base was glued on an aluminum holder, which contained a heating element, a temperature sensor, and a set of distance detectors. The DPO holder rested on a passive vibration isolation stage made of alternating steel disks and silicon gel dampers. The use of this vibration isolation system allowed us to strongly reduce the influence of external disturbances on the oscillator, e.g., seismic noise. A piezoceramic actuator, mounted on the lowest stage of the vibration isolation system, was used for excitation and permitted diagnostics (determination of resonance frequency and Q-factor) and sensitivity studies. The apparatus was operated in a vacuum chamber at a pressure of about 10^{-7} mbar. The pressure in the vacuum vessel was constantly monitored during the complete duration of the measurement, since its variation could have induced a change in the Q-factor of the DPO.

The detection of the angular deflection of the DPO's head was performed by an optical lever. It consisted of a He-Ne laser beam that was reflected by the oscillator head onto a position-sensitive (split) photodiode. The resolution of this detection system was about 3×10^{-11} rad in a 1 Hz measurement bandwidth. As shown in our previous work,[26] the stabilization of the DPO temperature is necessary in order to minimize oscillator frequency drift and thus maximize the effects of an external constant-frequency excitation. Using a PID controller, we reduced the temperature instability to the level of 0.08 K over several hours, which corresponds to a resonance frequency instability of about 0.01 Hz.

The measurements analyzed below were taken in several consecutive runs for a total of 3.4×10^5 s. The detection of the oscillator's angular displacement was performed by a digital dual-phase lock-in amplifier, with a local oscillator frequency set to allow measurement of the quadratures $X(t)$ and $Y(t)$. The bandwidth of the lock-in amplifier was set equal to 0.8 Hz (corresponding to a lock-in time constant of $\tau_L=0.3$ s) and the data were acquired at a rate of 1 Hz. A run was divided into blocks where each block consisted of the following steps. First, the oscillator response was detected during 600 s, while an external excitation was applied. The external excitation was then switched off and, after waiting 100 s, necessary for the oscillator to reach equilibrium, a measurement without excitation was taken over another 600 s. Then, the (slowly drifting) resonance frequency was determined by exciting the DPO with a fixed voltage at a few different frequencies and fitting the measured amplitude to a Lorentz curve. Once this procedure was completed, the resonant excitation was modified, if necessary, and the next block started.

In order to determine the influence of the detection system noise, the quadratures $X(t)$ and $Y(t)$ were also measured in the absence of external excitation and with the local oscillator frequency tuned 1 Hz below the DPO resonance. These measurements were performed in a single run with a duration of 2×10^5 s.

B. Characterization of thermal noise

In order to characterize the measured oscillator deflection noise, we first analyzed the data taken in the absence of external excitation. These data were considered as taken all in a single measurement without dead time between the single runs.[27] Figure 2 displays the statistics of the X quadrature for two different acquisition times, 2500 s and 1.7×10^5 s. In both cases the experimental data were fitted to Eq. (6), with the exponents as fit parameters. The plots of the fit residuals show how the agreement with theory improves for increasing measurement time, as expected. Also, the noise of the detection system was found to be approximately 50 times smaller than the thermal noise of the DPO, using the procedure described above. Assuming the statistics to be indeed due to Brownian noise, from the fits it is possible to obtain the torsion constant of the oscillator, $D=(8.04\pm0.06)\times10^{-2}$ N m.

The torsion constant can also be calculated approximately from the oscillator's dimensions and is given by[28]

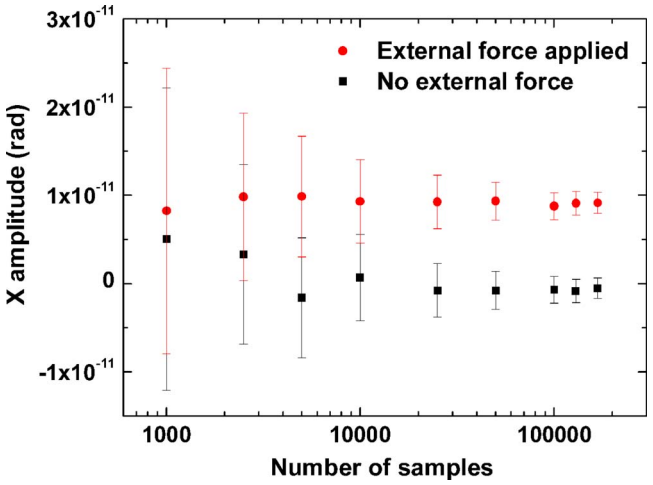

FIG. 3. The mean values of the oscillator's X quadrature, measured with and without a small external mechanical excitation (4.3×10^{-18} N m), as a function of integration time. Each sample corresponds to 0.3 s measurement time. The shown error bars correspond to ±3 standard deviations of the mean values of the individual lock-in readings.

$$D=\xi\frac{ab^3}{c}G, \tag{7}$$

where ξ is a parameter depending on the geometry of the oscillator, equal to 0.25 in our case, $a=6.85$ mm is the full width of the oscillator's head, $b=0.3$ mm is the thickness, and $c=1.09$ mm is the neck width. The calculated torsion constant is $D=7.7\times10^{-2}$ N m, in good agreement with the value obtained from the noise data. This confirms that the observed noise is Brownian noise.

C. Detection of weak torques

In order to determine the torque sensitivity of the sensor, a small harmonic excitation was applied to the oscillator, as previously described. This was implemented by applying a small ac voltage at the sensor's resonance frequency to the piezoceramic actuator mounted on the vibration isolation system. The excitation voltage was generated by a frequency synthesizer phase-locked to the local oscillator used for the lock-in detection. In order to determine the correspondence between voltage and torque, the excitation was made large enough to produce an easily detectable deflection, which was converted into a torque value by multiplying it with the experimentally determined spring constant. In doing so, we made sure that the piezo actuator's response was linear in the range used. In the following we used an external excitation corresponding to a torque $\Gamma_0=4.3\times10^{-18}$ N m and its phase was chosen equal to the local oscillator phase. According to the theory of Sec. II, this torque should be detectable for integration times exceeding approximately 1×10^4 s.

Figure 3 shows the mean values of the X quadrature as a function of the averaging time. Note that the mean of the lock-in measurements is the time average of the Fourier amplitude of X at the DPO's resonance frequency. In calculating these mean values we assumed that the dead times between successive runs do not introduce any deviation, in analogy to the case illustrated in the previous section. Since each sample was taken over 0.3 s, the "true" total integration time corresponds to 5.1×10^4 s. The shown error bars are equal to ±3 standard deviations of the mean value, calculated from the individual data points. The mean quadrature amplitude corresponds to, after subtracting the detection system noise, an excitation torque of 4.4×10^{-18} N m, in good agreement with the expected level.

For comparison, the figure also displays the mean X quadrature in the absence of mechanical excitation. As can be seen, the presence of an external excitation is masked by noise for averaging times shorter than 7×10^3 s, whereas it is visible for longer averaging, in accordance with the above estimate. Statistical testing was done to determine if the presence of the external signal resulted in a significant difference of the two sets of data. The result of a t-test for 2.5×10^4 samples indicated that the mean quadratures are statistically different with a significance level of 95%.

From the experimental data, we can estimate the minimum detectable torque as follows. The full data set is divided into 20 equally long subsets. The standard deviation of the subset mean values may be identified with the thermal amplitude noise, 1.3×10^{-12} rad. As a criterion for the minimum detectable torque we consider the torque equivalent to twice this deflection noise value, 1.3×10^{-18} N m. This holds for an integration time of 2.5×10^3 s. Extrapolation to an integration time of 5.1×10^4 s (the whole data set length), yields 2.8×10^{-19} N m. The theoretical value for this quantity, from Eq. (5) corrected by a factor $1/\sqrt{2}$ for the case of detection of a single quadrature, is 1.3×10^{-18} N m, a factor of 5 larger than the extrapolated experimental value. The origin of this difference is unclear.

IV. SUMMARY

We have shown that it is possible to detect weak torques on the order of few 10^{-18} N m by using a macroscopic single-crystal oscillator (sensitive area 12.5 mm^2), which can easily be fabricated in clean-room facilities. Moreover, we have experimentally reached the thermal-noise limited sensitivity of the detector. In particular, we showed that the measured noise level is in general agreement with Brownian noise theory. Thus, our apparatus represents a suitable approach for the detection of gravity-like new forces at short distances. Moreover, the sensor could also be employed for the detection of classical, e.g., magnetic forces.

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