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TESTING THE FOUNDATIONS OF RELATIVITY USING CRYOGENIC OPTICAL RESONATORS

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We present a new generation of experiments using cryogenic optical resonators (COREs) to test the foundations of relativity. The experiments test the isotropy of the speed of light (Michelson-Morley experiment), the independece of the speed of light from the velocity of the laboratory (Kennedy-Thorndike experiment), and the gravitational redshift for clocks based on an electronic transition. Compared with the best previous results, our tests have already yielded improvements up to a factor of three. Future versions promise significant improvements.

1. Introduction

The special theory of relativity (SR) and the principle of Lorentz covariance lie at the very foundation of today's understanding of physics. An experimental verification of the theory is therefore of great interest and is currently pursued by a new generation of ultra-high precision optical experiments, both space-based (OPTIS,[1] SUMO,[2] ...) and terrestrial. Examples for such tests are Michelson-Morley (MM) experiments, which test the isotropy of the speed of light $c(\theta) = c$, and Kennedy-Thorndike (KT) experiments, which test the independence of $c(v) = c$ on the velocity v of the observer relative to a presumed preferred frame of reference Σ; the natural candidate is the cosmic microwave background.

Tests of both Special and General Relativity are also motivated by the theoretical efforts to unify the forces of nature. Special Relativity, the principle of local position invariance (LPI), and the weak equivalence principle together make the Einstein Equivalence Principle (EEP), which General Relativity is based on. LPI implies the universality of the gravitational redshift, independent from the type of clock.

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No simple way to quantize gravity has been found. It is therefore believed that the principles underlying General Relativity may break down at a certain level of precision. Indeed, approaches towards a quantum theory of gravity have been put forward that lead to certain violations of EEP, for example loop gravity,[3,4,5] and string theory.[6,7,8] It is thus important to perform improved tests of SR and LPI.

According to common test theories,[9,10] SR follows unambiguously from MM, KT and Doppler shift experiments. Among these, MM-experiments currently offer the highest precision and are thus the most sensitive probe for possible violations of SR. KT-tests, on the other hand, are currently the least precise measurements and thus set the limit on the overall verification of SR. In this framework, deviations from SR can be parameterized as

$$\frac{c(v,\theta)}{c_0} = 1 + A\frac{v^2}{c_0^2} + B\frac{v^2}{c_0^2}\sin^2\theta\,, \tag{1}$$

where c_0 is the constant speed of light in the preferred frame Σ. The phenomenological parameters A and B vanish if SR is valid. Preceding the work described here, the best upper limits on these coefficients were $|B| < 5 \cdot 10^{-9}$ from a MM-experiment[11] and $|A| < 6.6 \cdot 10^{-5}$ from a KT-experiment.[12]

Both KT and MM-tests can be performed using cryogenic optical resonators (COREs), which provide a unique combination of extremely high long-term stability and no observable aging effects.[13,14] COREs can also be used to test the universality of the gravitational clock shift and thus the principle of local position invariance (LPI)[15], an essential foundation of the general theory of relativity.

The basic principle (Fig. 1) of all these experiments is to compare the oscillation frequencies of two clocks, ν_{I_2} (e. g. iodine-stabilized laser) and ν_{CORE} (CORE system), over a sufficiently long time. The dimensionless ratio of frequencies depends on the speed of light c and is also a function of fundamental constants like the fine structure constant α.[16] The experiment measures this ratio over time and is therefore sensitive to possible dependencies on the spatial orientation, the laboratory velocity or the gravitational potential (schematic shown in Fig. 2).

2. Kennedy-Thorndike experiment

Our KT experiment[18] makes use of the exceptional features of COREs and yields a 3-fold improved accuracy compared with the best previous experiment by Hils and Hall.[12] We compared the frequencies of a CORE-stabilized laser at 1064 nm and an absolute iodine frequency standard (Fig. 3) over a period of 190 days (Fig. 4). The frequency of such a laser is $\sim c/l$ (resonator length: l) and is thus sensitive to a possible dependency of c on the velocity of the laboratory v, which is given by solar system motion.

Due to the remarkable long term stability and absence of creep in COREs (drift <20 Hz/day,[14] and <0.1 Hz/day in CORE/CORE comparisons[19]), it was thus for the first time possible to access the large annual variation of ±30 km/s in v due to the

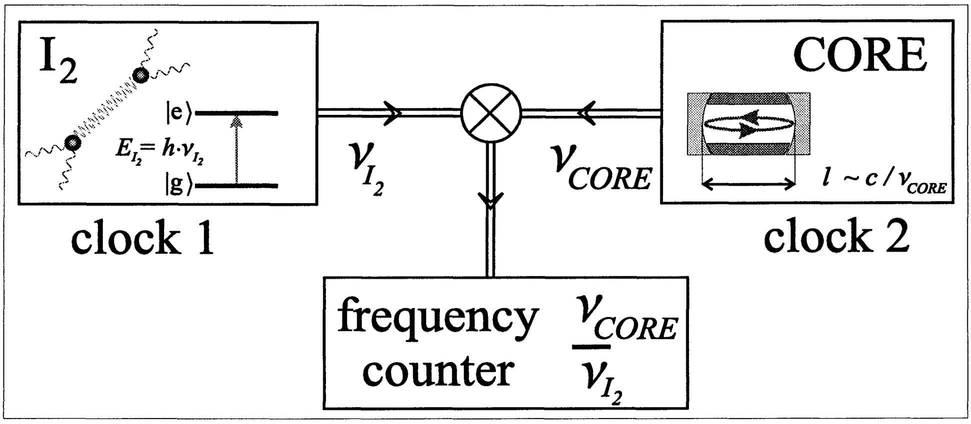
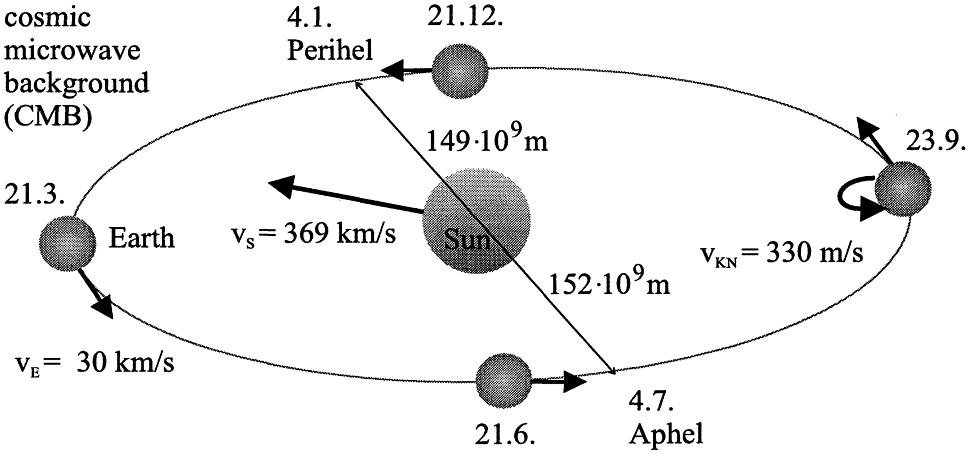


Fig. 1. Principle of fundamental tests of physics using clocks.



$$v = v_S + v_E \sin\{\omega_E(t - t_o)\} \cos 11^\circ + v_{KN} \sin\{\omega_{KN} t\} \cos 7^\circ$$

$$\Phi(t) = -3GM/2a - \underbrace{2GMe/a}_{6.7\cdot10^{-10}c^2} \cdot \cos\{\omega_E(t - t_E)\} - \underbrace{GMr/a^2}_{2.46\cdot10^{-13}c^2} \cos\{\omega_{KN}\, t\} \cos(\phi_{in}) \sin(\phi_{lat})$$

Fig. 2. Relevant velocities and gravitational potentials for fundamental tests utilizing motions within the solar system. G is the gravitational constant, M the solar mass, a the semi-major axis of Earth's orbit, e it's eccentricity, r Earth's radius, $\phi_{inc} \sim 23.5^\circ$ the inclination of Earth's rotation axis, and $\phi_{lat} = 47.4^\circ$ the latitude of Konstanz. The direction of the Sun's velocity relative to the cosmic microwave background according to COBE data is $(11^h 11^m 57^s \pm 23^s, -7^\circ.22 \pm 0^\circ.08)$ in celestial coordinates.[17]

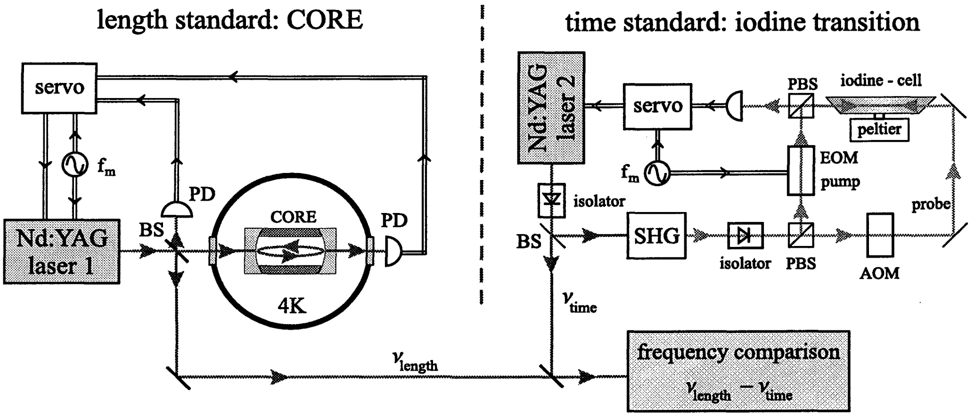


Fig. 3. Set-up of the Kennedy-Thorndike experiment.

Earth's orbit around the sun for a KT experiment. Previous resonator experiments, limited by the comparatively large creep of ULE resonators, could only make use of the much smaller daily variation of ±300 m/s due to Earth's rotation.

Following the phenomenological ansatz in Eq. 1 and applying the standard line of reasoning, we obtain

$$|A| = (1.9 \pm 2.2) \cdot 10^{-5}, \tag{2}$$

as compared to the best previous experiment,[12] which yielded $\pm 6.6 \cdot 10^{-5}$. Since in the framework of common test theories the accuracy of the KT-experiment is the limiting factor, this result *improves the overall accuracy of the verification of SR by a factor of 3.*

For unambiguous interpretation of MM- and KT-experiments it is also important to consider the influence of a spatial anisotropy on solid state properties. Since binding forces in atoms and solids are electromagnetic, modified Maxwell equations describing an anisotropic speed of light modify solid-state properties, like length, and also the frequency of atomic clocks. We derived a test theory including this influence.[20,21] For most experiments, including our own, it predicts an increased sensitivity to possible violations of SR.

Our experiment was clearly limited by the performance of the iodine standard. A new version will replace it with a femtosecond optical comb generator[22,23] directly locked to a primary cesium clock or hydrogen maser. The sensitivity should thus improve by at least one order of magnitude with no assumed further improvements in CORE performance.

3. Test of local position invariance

Assuming SR is correct, we may give an alternative interpretation of this measurement as a test of the principle of equivalence by a null gravitational red-shift experiment, and thus as a test of the Einstein equivalence principle (EEP) of gen-

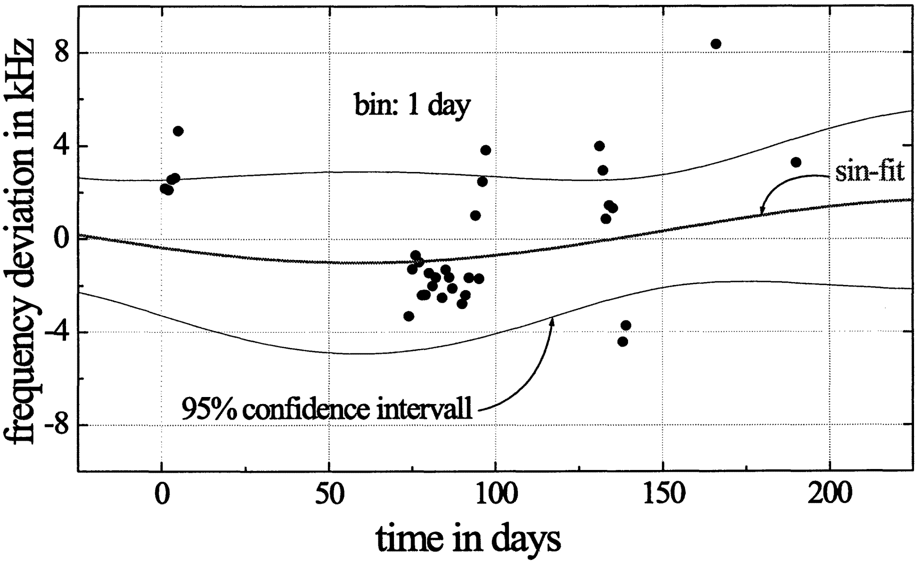


Fig. 4. 190 day beat measurement (starting October 10th, 1997) between an iodine standard and CORE. 1-day averages are shown. Earth's annual motion around the sun would lead to sinusoidal frequency variations if $c(v) \neq$ const. No variations were found within ±1500 Hz. The 95 % confidence interval of a possible 1-year variation, including an uncertain offset, is shown.

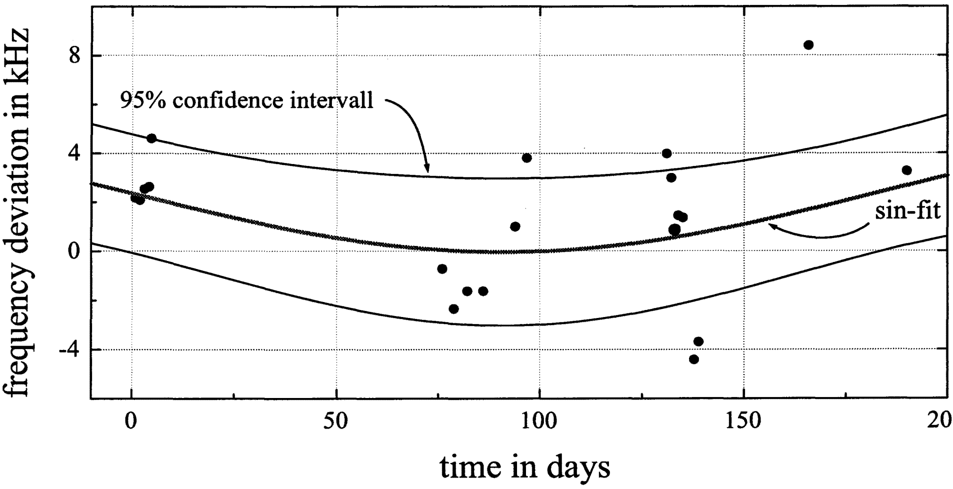


Fig. 5. The same data analyzed for a possible LPI violation. Several data points from the cluster between $t = 75$ and $t = 100$ days were removed to avoid an imbalance of statistical weights.

eral relativity. The gravitational red-shift of spectral lines is a direct consequence of EEP. The following phenomenological ansatz can be made for a possible dependency of two different clocks I_2 and $CORE$ within the varying gravitational potential $\Phi(t)$:

$$\frac{\nu_{I_2}}{\nu_{CORE}} - 1 = (\zeta_{I_2} - \zeta_{CORE}) \cdot \Phi(t)/c^2, \tag{3}$$

$\zeta_{I_2,CORE}$ are dimensionless parameters that measure the LPI violation of the considered clocks. Both ζ_{I_2} and ζ_{CORE} vanish if the EEP is valid. A dependence of the frequency ratio ν_{I_2}/ν_{CORE} on $\Phi(t)$ would imply $\zeta_{I_2} \neq \zeta_{CORE}$, a redshift dependent on the nature of the clock — in contrast to the universality predicted by the EEP.

From the measurement shown in Fig. 5, we obtain an upper limit of (2.4 ± 1.4) kHz for a possible yearly frequency variation. Inserting the yearly variation (see Fig. 2) of the Sun's gravitational potential $\Phi(t)/c^2$ with an amplitude of $3.4 \cdot 10^{-10}$ (a daily variation of the potential is smaller by a factor of 1000), we obtain

$$|\zeta_{CORE} - \zeta_{I_2}| < (2.2 \pm 1.5)\% \tag{4}$$

for the LPI violation parameters. We regard this as a confirmation of LPI within the experiment's accuracy, comparable with the result of an analogously performed experiment.[24] The combination of these two experiments provides the first limit to the universality violation of the redshift of an electronic transition at a 4% level of accuracy. The best overall confirmation of LPI is at present still provided by comparisons of atomic frequency standards.[25]

4. Michelson-Morley experiment

The same experimental setup employed for the Kennedy Thorndike experiment could in principle also be used for a Michelson-Morley experiment to test the isotropy of space. In this case, one would analyze the data for a possible violation signal with a period of 12 hours (compared to the 24 hours for the KT-experiment). The alternative setup using two orthogonal cavities as shown in Fig. 6, however, offers superior performance since it is not limited by the stability of the Iodine frequency standard. It also doubles the amplitude of a hypothetical violation signal. To perform a measurement, one can once again rely on Earth's rotation. This is only possible due to the ultra-high long-term stability of COREs, which also avoids the problems of previous experiments which were plagued by the relativley large creep of ULE resonators.

Denoting the frequencies of the two cavity-locked lasers by ν_1 and ν_2 and their beat frequency change by $\Delta\nu(\theta) = \Delta\nu_1(\theta) - \Delta\nu_2(\theta)$, the isotropy violation signal is

$$\frac{\Delta\nu(\theta)}{\nu} \approx \frac{\Delta\nu_1}{\nu_1} - \frac{\Delta\nu_2}{\nu_2} = \frac{c(v,\theta)}{c_0} - \frac{c(v,\theta+\pi/2)}{c_0} = -B\frac{v^2}{c_0^2}\cos 2\theta \tag{5}$$

using the coefficient B from Eq. 1. A current implementation of such a setup based on existing components is shown in Fig. 6. The laser beams are coupled to the

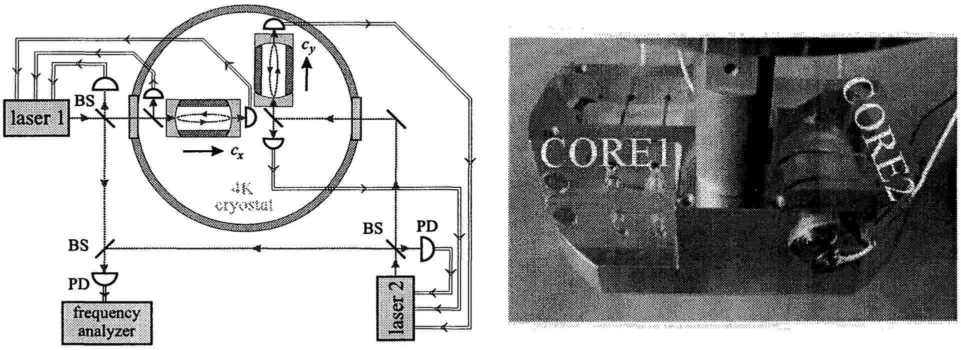


Fig. 6. Setup for a Michelson-Morley experiment for testing the isotropy of space (left) and photo of a gold plated copper mount holding two COREs (right). This mount provides some cancellation of thermal effects by coupling both COREs to the same thermal bath.

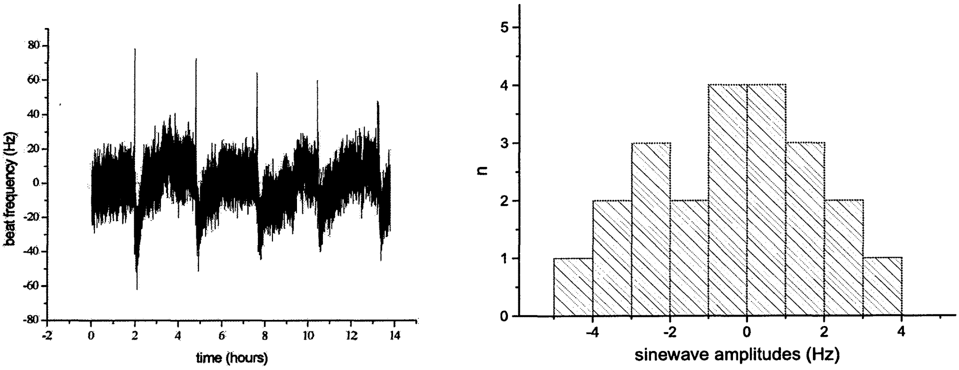


Fig. 7. Left: Beat frequency between two orthogonal COREs. Typical data block after subtraction of linear drift. The periodic spikes are due to the liquid nitrogen refills. Right: Histogram of possible violation amplitudes from fits of 22 datasets.

cavities through optical windows and the lasers are locked in reflection using the Pound-Drever-Hall method with $3\,f_m$ detection. The powers transmitted through the cavities are at a level of a few 100 nW. The free-space coupling of the two beams implies that a relative shift of the beams (for example caused by refills of liquid Helium or Nitrogen) will lead to shifts in the locked frequency due to spurious etalons.

Beat data was collected during nights and week-ends over a period spanning 6 weeks, so individual data blocks are typically half a day to a few days long. For each data block, we remove the linear drift and fit the expression Eq. (5) with the angle $\theta = (t - (10\,\mathrm{h}\,34\,\mathrm{min}))/12\,\mathrm{h}$ on August 2nd, 2001. An example is shown in Fig. 7. Analyzing 22 records we obtain the histogram shown in Fig. 7 and we find a beat modulation amplitude of (-0.39 ± 0.58) Hz, leading to

$$|B| = (8.7 \pm 13) \cdot 10^{-10}. \tag{6}$$

This implies a 3-fold improvement compared to the best previous test,[11] which yielded $|B| < 5 \cdot 10^{-9}$. Improving the current setup and taking data for a sufficiently long time, it should be possible to achieve an improvement by another factor of 3 using the current setup. Further improvements are expected for a new, actively rotated setup using specially designed monolithic COREs.[26]

5. Summary

In conclusion, we have performed experiments using COREs which set a new experimental limit on violations of Special Relativity and give a confirmation for validity of the Einstein equivalence principle. Compared to the experiment of Hils and Hall,[12] our Kennedy-Thorndike experiment provides a 3–fold improved limit on a possible velocity dependence of the speed of light. Since according common test theories,[9,10] the present verification of SR is limited by the Kennedy-Thorndike test, this limit also provides an improvement of the overall validity of Special Relativity by a factor of 3. Compared to the experiment of Brillet and Hall[11] first data from our Michelson-Morley experiment already represents a 3–fold improved test of the isotropy of space. Within the common test theories, this is to our knowledge the highest accuracy optical test of Relativity. Future improvements by several orders of magnitude seem possible.

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